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# Hydrodynamic Effects in Colliding Solids

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## INTRODUCTION

Recently there has been growing interest in the study of phenomena arising in the collision of solids (metals or rocks) with velocities of the order of several hundred m/sec. The investigation of oblique collisions, when the moving solid impacts at an angle with the contact surface, is of particular interest. The phenomenon of wave-formation occurring in these cases is very amusing and useful from the point of view of explosive welding of metals [1]. Periodic waves are observed on the collision surface when the velocity at contact does not exceed the sound velocity in the metals. The effect of wave formation was first described in [2]. Experiments have shown that the amplitude and period of the waves are determined by the parameters describing the collision [3], [4]. A qualitative explanation of the process of wave formation has been attempted [5], [6]. In a recent paper [7] a quantitative theoretical explanation of the wave formation was described. However, this paper is unsatisfactory from our point of view owing to the arbitrariness of the basic hydrodynamical model and the vagueness of the simplifying assumptions. The author of [7] pointed out the discrepancy between his theory and the experiments of [6] and this discrepancy is confirmed by our investigations. The purpose of the present paper is to analyze some phenomena essential, in our opinion, to the understanding of oblique collision. Postponing the detailed investigation of the mechanism of wave formation itself, we shall limit ourselves to the analysis of a number of accompanying effects, for instance, the causes of the formation of waves, theoretical calculations of long-wave oscillations modulating the usual short waves and some other effects. The calculations showed that the solution of a corresponding hydrodynamical problem is described satisfactorily by a linear acoustical model containing the characteristic length for the wave formation process.

## 1. Statement of the Problem in the Acoustical Approximation

The process of oblique collision of plates can be considered in the following way. The flow will be assumed steady for sufficiently long plates, in a coordinate system with its origin at the point of contact. Let us assume that there is no vertical velocity at large distances from the origin (Fig. 1). Here $\gamma_1$ and $\gamma_2$ are the angles of

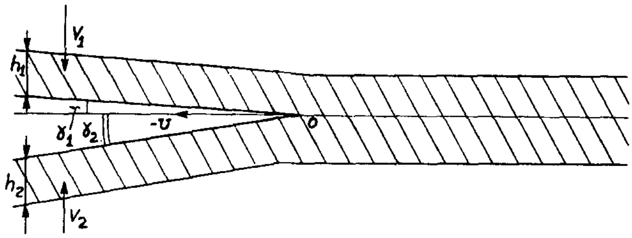


Figure 1

inclination of the plates to the $x$ axis, $U$ is the velocity of contact, $h_1$ and $h_2$ are the thicknesses of the plates, $V_1$ and $V_2$ are the velocities of the plates, and $\gamma = \gamma_1 + \gamma_2$ is the angle of collision. The velocities are connected by the relations:

$$| V_1 |/\sin \gamma_1 = | V_2 |/\sin \gamma_2 = U. \tag{1}$$

We shall consider the most important case of small angles $\gamma_1$ and $\gamma_2$, i.e., $| V_1 |/U \ll 1$; $| V_2 |/U \ll 1$. In this limiting case, the absence of a vertical velocity behind the point of contact corresponds to

$$\rho_1 h_1 | V_1 | = \rho_2 h_2 | V_2 | \tag{2}$$

since higher order terms can be neglected. Here $\rho_1$ and $\rho_2$ are the densities of the materials in the upper and lower plates, respectively.

The equations of motion in the acoustic approximation are:

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} &= 0 \\ \frac{\partial v}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial y} &= 0 \\ \frac{\partial p}{\partial t} + \rho c^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) &= 0. \end{aligned} \tag{3}$$

Here $u$ and $v$ are the components of velocity in the $x$ and $y$ direction, $p$ is the pressure, $\rho$ is the density at $p = 0$, and $c$ is the sound velocity. In accordance with the assumption of steady flow, relative to the origin, we shall determine the solution of Eq. (3) in the form:

$$\begin{aligned} u &= U + u'(x + Ut, y) \\ v &= v'(x + Ut, y) \\ p &= p'(x + Ut, y). \end{aligned} \tag{4}$$

We shall assume that:

$$U < c. \tag{5}$$

In the coordinate system $x'$, $y'$ which moves with the point of contact, Eq. (3) is transformed to

$$\begin{aligned} U \frac{\partial u'}{\partial x'} + \frac{1}{\rho} \frac{\partial p'}{\partial x'} &= 0 \\ U \frac{\partial v'}{\partial x'} + \frac{1}{\rho} \frac{\partial p'}{\partial y'} &= 0 \\ U \frac{\partial p'}{\partial x'} + \rho c^2 \left( \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} \right) &= 0. \end{aligned} \tag{6}$$

The primes will be omitted henceforth.

From the first equation of (6):

$$u = -(p/\rho U) + \mu(y). \tag{7}$$

Since it is possible to assume that $u = 0, p = 0$ at a large distance to the left of the point of contact, we can put $\mu(y) \equiv 0$ in Eq. (7). However, it is necessary to take into account the fact that $\mu(y) \neq 0$ on the right side of the point of contact, if the effects of dissipation, causing vorticity formation, are to be included. This is impossible to do within the limits of our model. In fact, we obtain from the first two equations of (6), after differentiation and subtraction:

$$\frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) = 0 \quad \text{or} \quad \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0.$$

Since the flow is irrotational on the left side of the point of contact, it is easy to obtain from Eq. (7): $\partial\mu/\partial y = \partial u/\partial y - \partial v/\partial x = 0$; and therefore $\mu \equiv$ const. everywhere in the flow, i.e., $\mu \equiv 0$ everywhere. Therefore, the pressure from Eq. (7) is

$$p = -\rho U u. \tag{8}$$

Eliminating $p$ from Eqs. (6) results, after transformation, in

$$\begin{aligned} &\frac{\partial u}{\partial x} + \frac{\partial}{\partial(y\sqrt{1 - U^2/c^2})}\left[\frac{v}{\sqrt{1 - U^2/c^2}}\right] = 0 \\ &\frac{\partial u}{\partial(y\sqrt{1 - U^2/c^2})} - \frac{\partial}{\partial x}\left[\frac{v}{\sqrt{1 - U^2/c^2}}\right] = 0. \end{aligned} \tag{9}$$

Eq. (9) shows that the function $f(z)$, determined by,

$$f(z) = u - i\frac{v}{\sqrt{1 - U^2/c^2}}, \qquad z = x + iy\sqrt{1 - U^2/c^2} \tag{10}$$

is an analytical function in the flow region. To determine $f(z)$, boundary conditions must be imposed. If the two colliding plates have different densities, the values of $f(z)$ and $z$ differ for each plate.

## 2. Solution for Identical Densities

Let $\rho_1 = \rho_2 = \rho_0$ and $c_0$ be the sound velocity of the colliding plates. The point of contact is at the origin. The equation for the boundary of the upper plate is

$$\operatorname{Im} z = \sqrt{1 - \frac{U^2}{c^2}}\, h_1$$

and, correspondingly, for the lower plate:

$$\operatorname{Im} z = -\sqrt{1 - \frac{U^2}{c^2}}\, h_2 .$$

Assuming the plates to be attached after collision, $f(z)$ is analytic everywhere in the region

$$-h_2\sqrt{1 - U^2/c_0^2} \leqslant \operatorname{Im} z \leqslant h_1\sqrt{1 - U^2/c_0^2}$$

except at the origin.

The cut along $x < 0$, $y = 0$ corresponds to the free edge of the plates. $p = 0$ on the free boundaries including the cut. From Eq. (8) we have $u = 0$ together with $p = 0$.

The situation in the physical plane is shown in Fig. 2. The conditions at infinity are:

$$
\begin{array}{lll}
u = 0 & \text{as} & x \to \pm\infty \\
v = 0 & \text{as} & x \to +\infty \\
v \to -|V_1| & \text{for} & y > 0 \quad x \to -\infty \\
v \to |V_2| & \text{for} & y < 0 \quad x \to -\infty.
\end{array}
$$

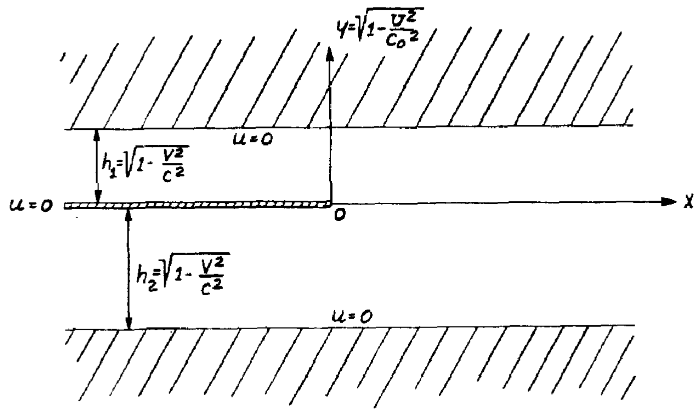


FIGURE 2

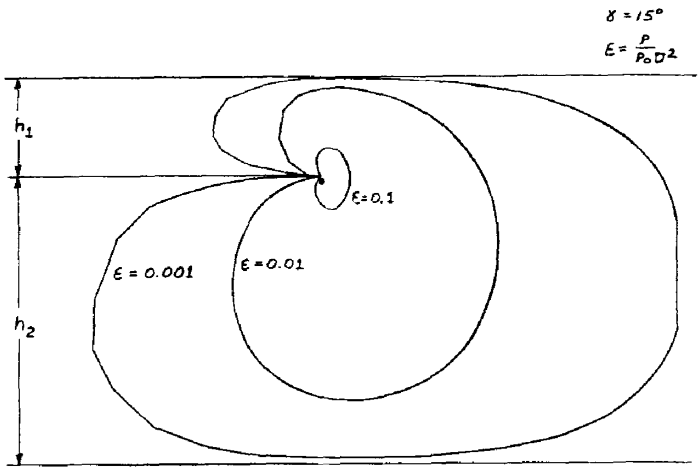


FIGURE 2a

The comparison of this model with the results of computation described below suggests the additional conditions for $f(z)$:

$$f(z) \sim \frac{\text{const}}{\sqrt{z}}$$

in the vicinity of $z = 0$.

It is easy to verify that all conditions will be met for $f(z)$ determined by

$$f(z) = iB/S, \qquad \operatorname{Im} S \geqslant 0. \tag{11}$$

Here $S$ and $B$ are defined by the relations:

$$\begin{aligned} z &= \frac{\sqrt{1 - U^2/c_0^2}}{\pi} \left\{ h_1 \ln \left(1 + \frac{S}{h_1}\right) + h_2 \ln \left(1 - \frac{S}{h_2}\right) \right\} \\ B &= \frac{h_1 \mid V_1 \mid}{\sqrt{1 - U^2/c_0^2}} = \frac{h_2 \mid V_2 \mid}{\sqrt{1 - U^2/c_0^2}} \end{aligned} \tag{12}$$

If $\rho_1 = \rho_2 = \rho_0$, it follows from Eq. (2) that $h_1 \mid V_1 \mid = h_2 \mid V_2 \mid$ and from Eq. (1) that

$$B = \frac{U}{\sqrt{1 - U^2/c_0^2}} \cdot \frac{2h_1h_2}{h_1 + h_2} \sin \frac{\gamma}{2}. \tag{13}$$

It is easy to check that the asymptotic form of $f(z)$ from (11), (12), and (13) is determined by:

$$f(z) \sim \frac{U \sin \gamma/2}{\sqrt[4]{1 - U^2/c_0^2}} \sqrt{\frac{2h_1h_2}{\pi(h_1 + h_2)}} \cdot \frac{1}{\sqrt{z}}. \tag{14}$$

It is possible to determine the radius of curvature of the free surface in the vicinity of the origin. It is the natural parameter, with the dimensions of length, which should be characteristic of the processes occurring in the neighborhood of the free boundary after collision, in spite of the act that our model is invalid near the point of contact.

The following equation gives the displacement of the free boundary:

$$\frac{\partial \delta}{\partial t} = v = \frac{A}{\sqrt{Ut - x}} \qquad \text{or} \qquad U \frac{\partial \delta}{\partial (x - Ut)} = \frac{A}{\sqrt{Ut - x}}. \tag{15}$$

Here $x$ is the fixed coordinate, and $A$ is a constant. Integrating Eq. (15) we have, since $\delta = 0$ at $x = 0$:

$$\delta = 2A/U \sqrt{\mid Ut - x \mid}$$

or in coordinates fixed at the point of contact:

$$\delta = 2A/U \sqrt{|x|}.$$

Therefore $\delta(x)$ is a parabola with the radius of curvature

$$R = 2A^2/U^2.$$

Substituting $A$ we obtain a formula for $R$:

$$R = \frac{2}{\pi}\sqrt{1 - \frac{U^2}{c_0^2}} \cdot \frac{2h_1 h_2}{h_1 + h_2} \sin^2 \frac{\gamma}{2}. \tag{16}$$

As $h_2 \to \infty$:

$$R = 2/\pi \sqrt{1 - U^2/c_0^2} \cdot 2h_1 \sin^2 \gamma/2.$$

The structure of this formula may be compared with the empirical relation determining the period of the waves in explosive welding [1]:

$$\lambda = 26h_1 \sin^2 \gamma/2.$$

The fact that the relations between $R$ and $\gamma$, and $\lambda$ and $\gamma$, are identical, suggests the development of wave formation in the vicinity of the point of contact with $R$ as a characteristic length. The verification of the dependence of $\lambda$ on $U$ in the form of

$$\sqrt{1 - U^2/c_0^2}$$

is experimentally difficult at present.

From Eqs. (11), (12), (8) the pressure can be calculated. If we know the velocity $U$ from experiment, and the size of the pressure zone of interest is fixed, we can calculate the minimum pressure and estimate the time of its action in this zone. This theory is unable to estimate the maximum pressure because the corresponding value of $t$ becomes infinite, but it it is possible to estimate the pressures in the layers of width greater than order of $R$. The isobar corresponding to the pressure $p_0$ is determined by:

$$x = \frac{\sqrt{1 - U^2/c_0^2}}{\pi}\left\{h_1 \ln \sqrt{\left(1 + \frac{\sigma}{h_1}\right)^2 + \frac{\zeta^2}{h_1^2}} + h_2 \ln \sqrt{\left(1 - \frac{\sigma}{h_2}\right)^2 + \frac{\zeta^2}{h_2^2}}\right\}$$

$$y = \frac{1}{\pi}\left\{h_1 \operatorname{arctg} \frac{\zeta}{\sigma + h_1} + h_2 \operatorname{arctg} \frac{\zeta}{\sigma - h_2} + (k_1 h_1 + k_2 h_2)\,\pi\right\} \tag{18}$$

$$\sigma = \pm \sqrt{\zeta(c - \zeta)}.$$

Here $c$ and $\zeta$ are determined by

$$c = \frac{\rho_0 U^2}{P_0} \cdot \frac{1}{\sqrt{1 - U^2/c_0^2}} \cdot \frac{2h_1 h_2 \sin \gamma/2}{h_1 + h_2}$$

$$0 \leqslant \zeta \leqslant c.$$

$k_1$ and $k_2$ are integers which are selected in such a way that the coordinate $y$ is continuous. Isobars are shown in Fig. 2a, for $c_0/U = 2$.

## 3. Validity of the Linear Model

The collision may be considered as the flow of an incompressible fluid in the case of $U \ll c$. The solution of the problem of the collision of jets is known [8]. The general features of the collision of two incompressible jets are shown in Fig. 3. Let $h_1$ and $h_2$ be the widths of the colliding jets at infinity, $h_3$ and $h_4$ are the widths of the jets after merging, $U$ is the velocity at infinity, $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ are the asymptotic angles of the jets with the $x$ axis. The transformation of the hodograph plane into a physical one is:

$$\begin{aligned} z = {} & \frac{h_1}{\pi\zeta_1} \ln\left(1 - \frac{\zeta}{\zeta_1}\right) + \frac{h_2}{\pi\zeta_2} \ln\left(1 - \frac{\zeta}{\zeta_2}\right) - \frac{h_3}{\pi\zeta_3} \ln\left(1 - \frac{\zeta}{\zeta_3}\right) \\ & - \left(\frac{h_1}{\zeta_1} + \frac{h_2}{\zeta_2} - \frac{h_3}{\zeta_3} - \frac{h_4}{\zeta_4}\right) \frac{\ln \zeta}{\pi} - \frac{h_4}{\pi\zeta_4} \ln\left(1 - \frac{\zeta}{\zeta_4}\right) \end{aligned} \tag{19}$$

when

$$\zeta_\kappa = e^{i\alpha_\kappa} \qquad \zeta = (u - iv)/U.$$

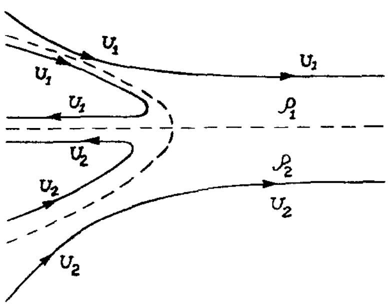


Figure 3

It is easy to obtain the following relations from the laws of conservation of mass and impulse

$$h_1 + h_2 - h_3 - h_4 = 0$$
$$\frac{h_1}{\zeta_1} + \frac{h_2}{\zeta_2} - \frac{h_3}{\zeta_3} - \frac{h_4}{\zeta_4} = 0. \tag{20}$$

When the angle of collision is small, i.e., $\alpha_4 \sim 0$, $\alpha_1 \sim 0$, $\alpha_2 \sim 0$, $h_3 \sim 0$ the situation in the hodograph plane is shown in Fig. 4. Near $\zeta = 1$ it is possible to put $\zeta = 1 + \zeta'$, where $\zeta'$ is small.

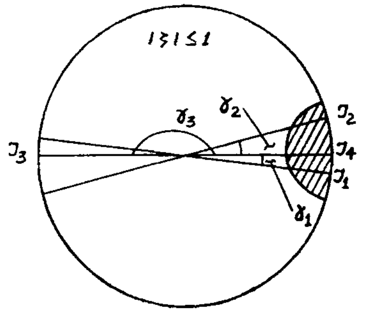


FIGURE 4

We shall get by transformation of Eq. (19):

$$z = \frac{h_1}{\pi}\ln(\zeta_1 - 1 - \zeta') + \frac{h_2}{\pi}\ln(\zeta_2 - 1 - \zeta') - \frac{h_1 + h_2}{\pi}\ln(-\zeta')$$
$$+ \left[ - \left(\frac{h_1}{\zeta_1} + \frac{h_2}{\zeta_2} - \frac{h_4}{\zeta_4}\right)\frac{\zeta'}{\pi} + \frac{h_1}{\pi\zeta_1}\ln\zeta_1 - \frac{h_2}{\pi\zeta_2}\ln\zeta_2 + \frac{h_4}{\pi\zeta_4}\ln\zeta_4\right].$$

From Eq. (20) it is possible to prove that the expression in the square bracket is small. We have from our previous definition $\gamma_1 = -\alpha_1$, $v - iv = U\zeta$ $\gamma_2 = \alpha_2$, so that

$$z = \frac{1}{\pi}\left\{h_1 \ln\left(1 - \frac{i \sin \gamma_1}{u - iv}\right) + h_2 \ln\left(1 - \frac{i \sin \gamma_2}{u - iv}\right)\right\}.$$

When $\gamma_1$ and $\gamma_2$ are small, using Eqs. (1) and (2), it is easy to get the expressions of Eqs. (11) and (12) for incompressible flow. This is the sought after result, the acoustical approximation is the asymptotic solution for the problem of collision of incompressible jets at small angles of collision.

## 4. The Case of Different Densities

Let two jets of incompressible fluid with different densities $\rho_1$ and $\rho_2$, collide with velocities $U_1$ and $U_2$ (Fig. 3). We have the condition at the contact boundary that

$$\left.\begin{aligned} p_1 &= p_2 \\ v_1/U_1 &= v_2/U_2 \end{aligned}\right\} \text{ i.e.,} \qquad \sqrt{\rho_1}\, U_1 = \sqrt{\rho_2}\, U_2 . \tag{21}$$

We shall introduce the variable $\zeta$:

$$\zeta = \sqrt{\rho}\,(u - iv),$$

where $\rho = \{\rho_1, \rho_2\}$, respectively, for the two zones of flow. The conditions of Eq. (21) mean that $\zeta(z)$ is an analytic function in the region of flow ($z = x + iy$).

We shall introduce the potential $\phi$ and the stream function $\psi$:

$$\frac{\partial \phi}{\partial x} = \sqrt{\rho}\, u; \qquad \frac{\partial \phi}{\partial y} = \sqrt{\rho}\, v; \qquad \frac{\partial \psi}{\partial x} = -\sqrt{\rho}\, v; \qquad \frac{\partial \psi}{\partial y} = \sqrt{\rho}\, u.$$

Evidently function $\Phi(z) = \phi + i\psi$ is an analytic function in the flow region except at the contact boundary. However, $d\Phi/dz = \sqrt{\rho}(u - iv) = \zeta(z)$ is analytic from Eq. (21) in the whole region of flow, and it follows that $\Phi(\zeta)$ is analytic in the hodograph plane.

Now we can find $z(\zeta)$ by standard methods:

$$z = \int \frac{d\Phi}{\zeta}.$$

Furthermore it is possible to get expressions similar to Eqs. (11) and (12) from the transformation of $z(\zeta)$ at small angles of collision:

$$z = \frac{1}{\pi}\left\{h_1 \ln\left(1 + \frac{S}{h_1}\right) + h_2 \ln\left(1 - \frac{S}{h_2}\right)\right\}$$

$$\operatorname{Im} S \geq 0$$

$$u' - iv' = \begin{cases} \dfrac{ih_1 \mid V_1 \mid}{S} & \text{at} \quad 0 < y < h_1 \\[2ex] \dfrac{ih_2 \mid V_2 \mid}{S} & \text{at} \quad -h_2 < y < 0. \end{cases}$$

Here it is necessary to assume that $h_1 \mid V_1 \mid \rho_1 = h_2 \mid V_2 \mid \rho_2$. The velocities are determined by the relations:

$$\begin{aligned} u - iv &= U_1 + u' - iv' \quad &&\text{at} \quad 0 < y < h_1 \\ u - iv &= U_2 + u' - iv' \quad &&\text{at} \quad -h_2 < y < 0. \end{aligned}$$

These formulae mean that it is possible to determine each jet separately by considering their interaction with a rigid wall.

It is convenient to introduce, following a similar procedure in Section 2:

$$h_1 \mid V_1 \mid = \frac{h_1 h_2 \rho_2}{\rho_1 h_1 + \rho_2 h_2} 2 \sin \frac{\gamma}{2} \cdot U_1$$

$$h_2 \mid V_2 \mid = \frac{h_1 h_2 \rho_2}{\rho_1 h_1 + \rho_2 h_2} 2 \sin \frac{\gamma}{2} \cdot U_2 .$$

The radius of curvatures of the free boundaries for the upper and lower plates are determined by:

$$R_1 = \frac{4}{\pi} \sin^2 \frac{\gamma}{2} \cdot \frac{h_1 h_2 (h_1 + h_2)}{(\rho_1 h_1 + \rho_2 h_2)^2} \rho_2^2$$
$$R_2 = \frac{4}{\pi} \sin^2 \frac{\gamma}{2} \cdot \frac{h_1 h_2 (h_1 + h_2)}{(\rho_1 h_1 + \rho_2 h_2)^2} \rho_1^2. \qquad (22)$$

In this case we obtain stable, asymmetric waves as in experiments with plates of different density (Fig. 5). Possibly, this experimental fact can be quantitatively explained by the asymmetry in the expressions of Eq. (22).

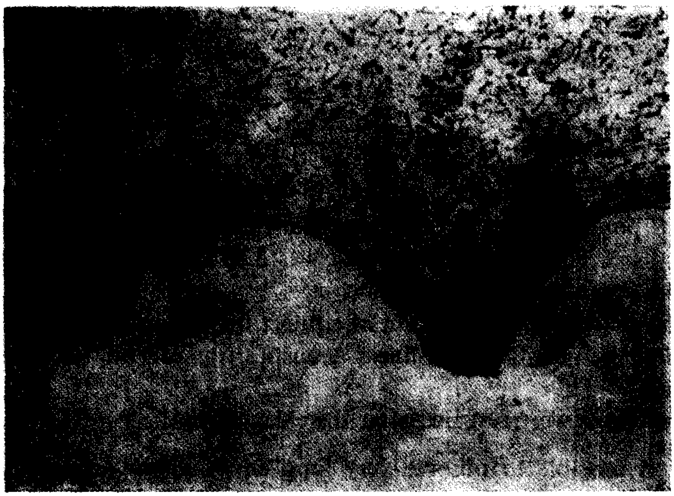

FIGURE 5

## 5. Calculations

We computed the problem of the collision of two plates with the program which solves two-dimensional equations of steady gas dynamics (Fig. 6). The upper plate was assumed to be copper ($\rho_1 = 8.93$ g/sm$^2$, $c_1 = 3.97$ km/sec), the lower to be iron ($\rho_2 = 7.87$ sm$^3$, $c_2 = 5.0$ km/sec). The angle of collision was assumed to be $\gamma = 10°$, the velocity $W_0$ to be 0.5 km/sec, and the size $h$ to be 20 cm. The implicit scheme was used in a moving net connected with the moving boundaries of the plates. The Courant criterion determines the time step to be of order $4 \times 10^{-8}$, therefore, the implicit characteristic of our scheme was not essential in this case.

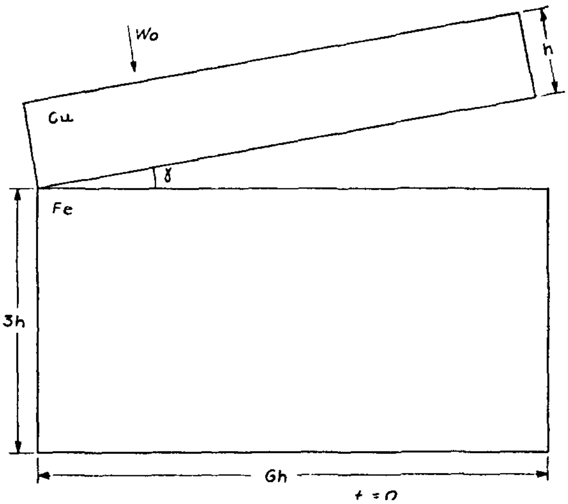


Figure 6

The net was uniform at the beginning of the process, 60 points in the horizontal direction, 15 points in the vertical one. The time-step was equal to $0.2 \times 10^{-6}$ $-0.4 \times 10^{-6}$ sec. The velocity of contact was 2.86 km/sec (subsonic). The distribution of the pressure at values of $x_1 \leqslant x_i \leqslant x_{28}$, $y_1 \leqslant y_s \leqslant y_1$ was plotted for 52 steps. The coordinate of the point of contact for this time was $x_{28}$ (Figs. 7–12). Calculation of the pressure from linear acoustics was carried out. The results are shown in Figs. 7–12.

Some new effects were revealed after the calculations.

(1) It is possible to observe very long and shallow waves (Fig. 13). The period of the waves exceeds many times the period of the wave determined by Eq. (17). Simultaneously, a wave of the same type was observed in experiments as

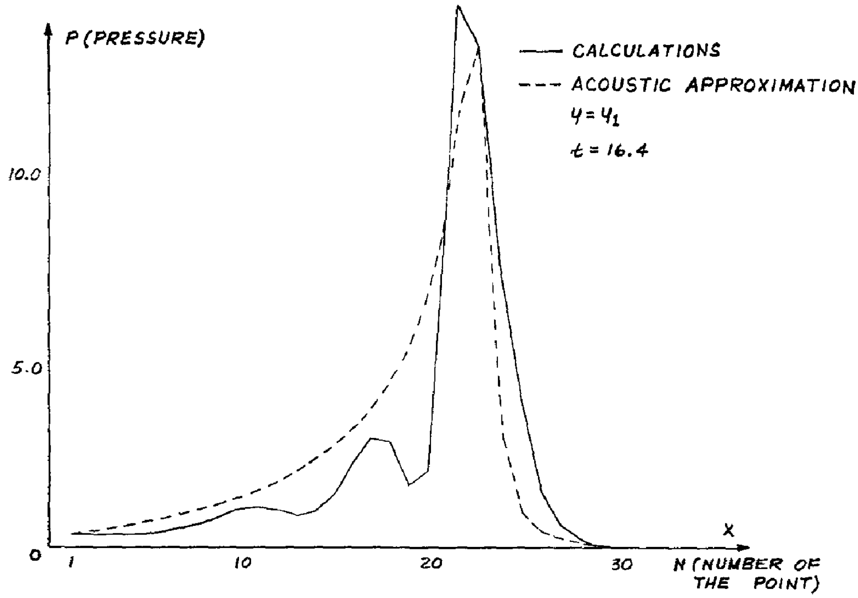


FIGURE 7

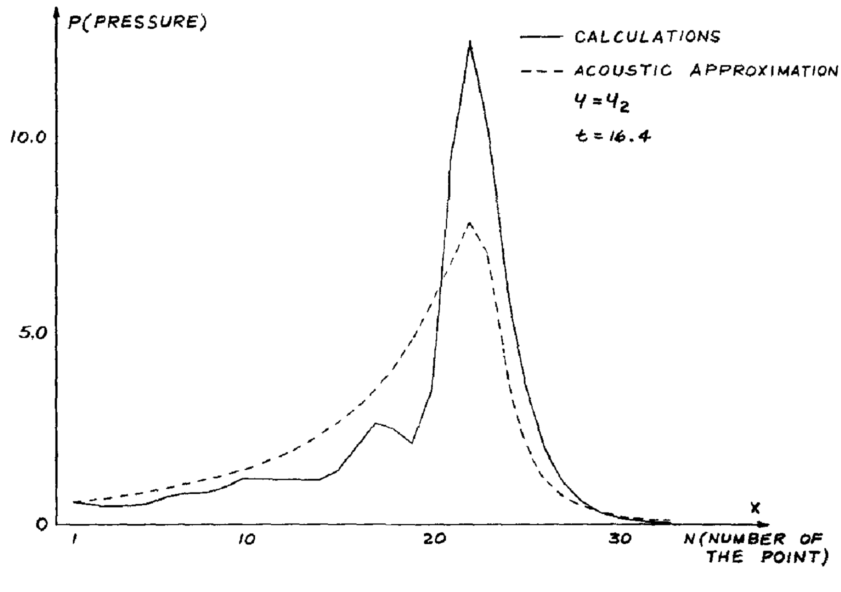


FIGURE 8

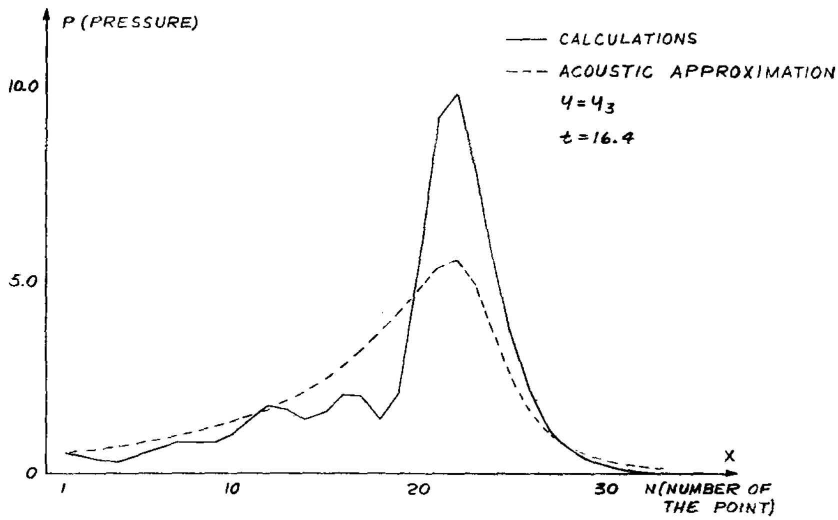


FIGURE 9

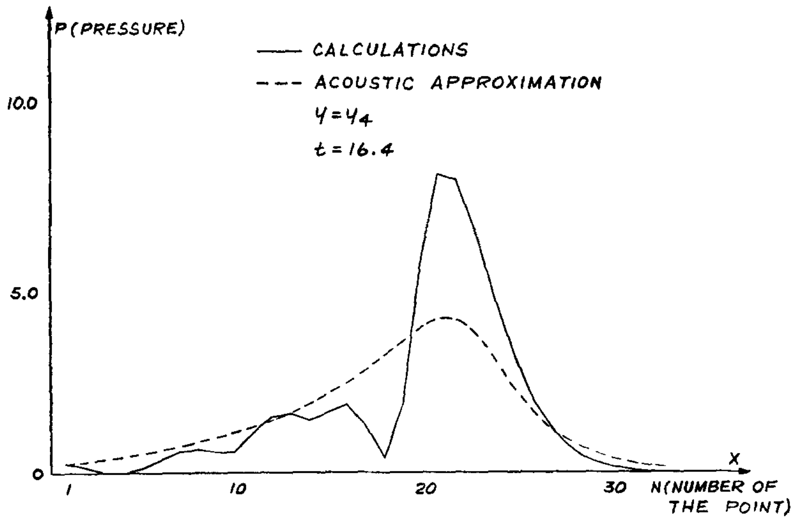


FIGURE 10

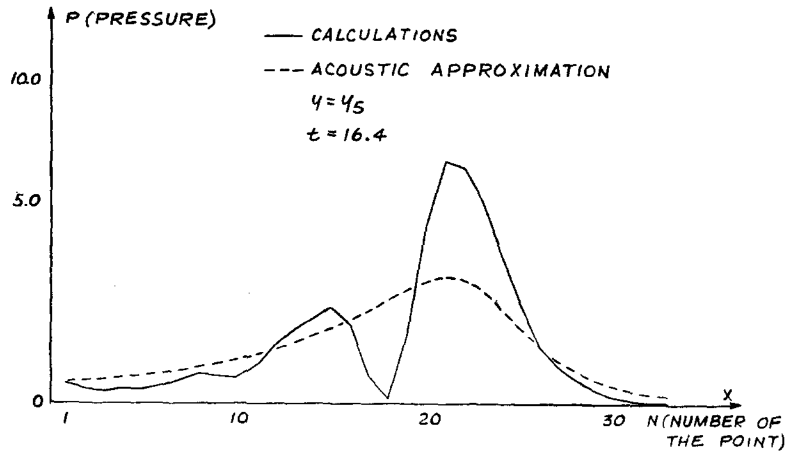


FIGURE 11

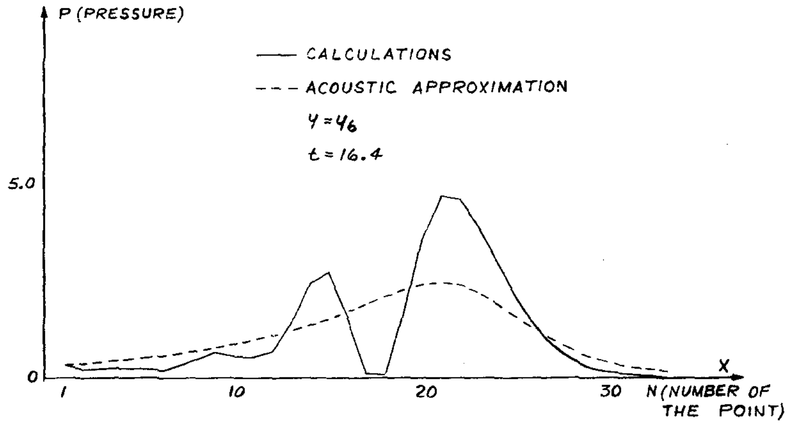


FIGURE 12

modulating the common waves (Fig. 14). The period of these waves is comparable with the calculated one. Let us note, that the half-period of the *long* waves is approximately equal to the time interval. This condition is necessary for the rarefaction wave reflected from the free boundary of the upper plate to overtake the point of contact.

The method of calculation of this interval is described below.

(2) The calculations showed the formation of a layer with low velocity near the surface of contact (intermediate jet). The parameters of this jet cannot be determined correctly owing to the fictitious viscosity of the scheme and the dependence of these parameters on the number of steps.

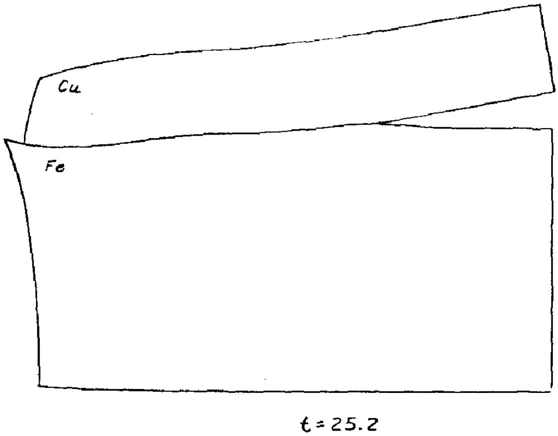


FIGURE 13

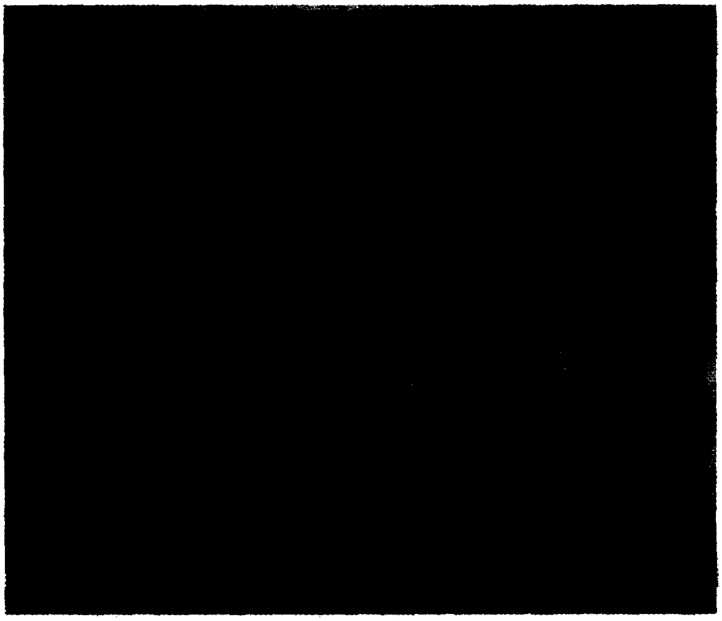

FIGURE 14

The calculations were carried out by the program for inviscid fluids. In Figs. 15, 16, 17, the variations of the horizontal velocity, the entropy and energy over the width of the plate are shown. These figures show flow of jet type.

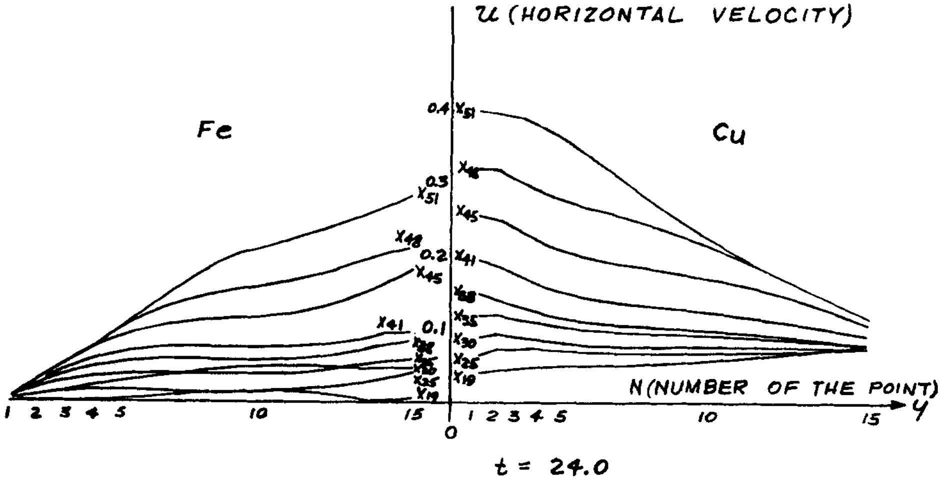


FIGURE 15

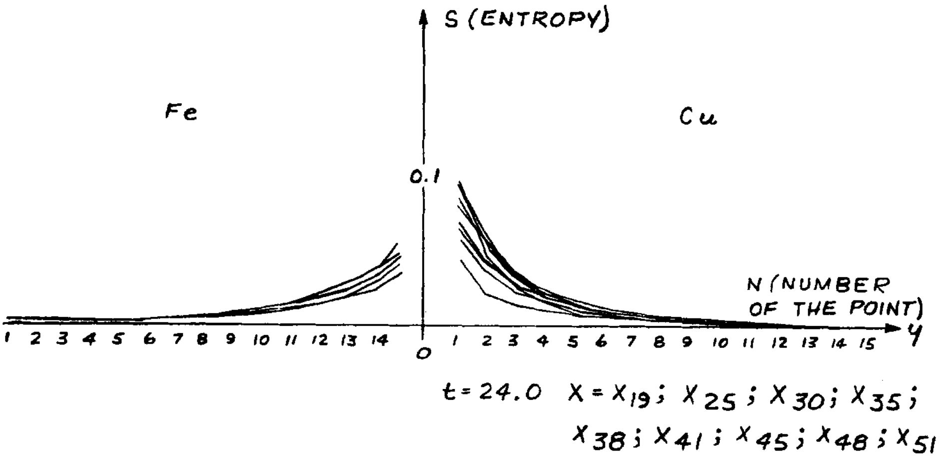


FIGURE 16

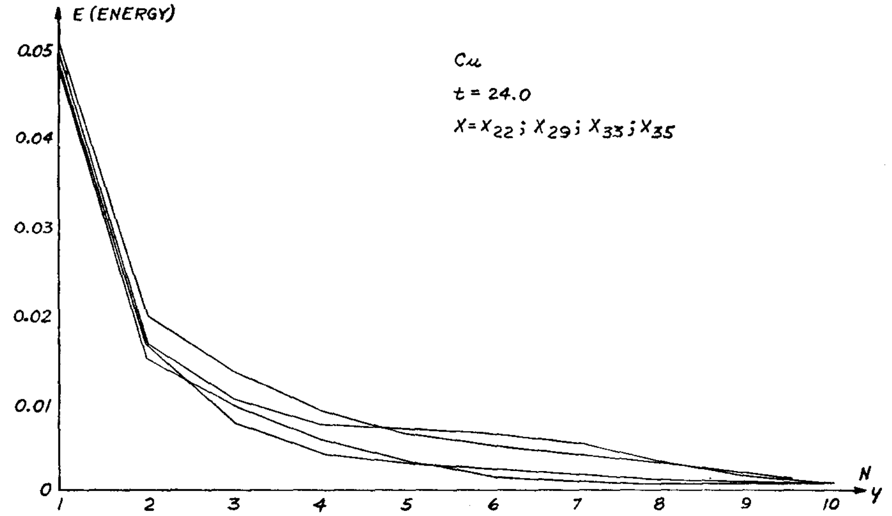


FIGURE 17

## 6. MECHANISM OF INITIATION OF WAVES

The experiments on the wave formation showed that the waves did not appear directly behind the point of contact, but at some distance from it. This experimental fact permits the assumption that the initiation of waves starts only after the emergence of the source of oscillations on the colliding surface. It is necessary to assume that the rarefaction wave reflected from the free boundary is this source of oscillation. This rarefaction wave emerges from the shock wave after collision (Fig. 18).

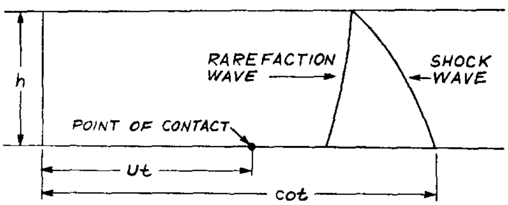


FIGURE 18

Let us assume for simplicity that the shock wave is weak and its velocity is equal to the sound velocity. Let the thickness of the upper plate be $h$ (Fig. 18). It is easy to calculate the time when the rarefaction wave on the contact surface and the point of contact coincide.

It is seen from Fig. 19 that

$$c_0^2t_0^2 = Ut_0^2 + 4h^2.$$

The moment when oscillations begin is determined by

$$t_0 = \frac{2h}{\sqrt{c_0^2 - U^2}}.$$

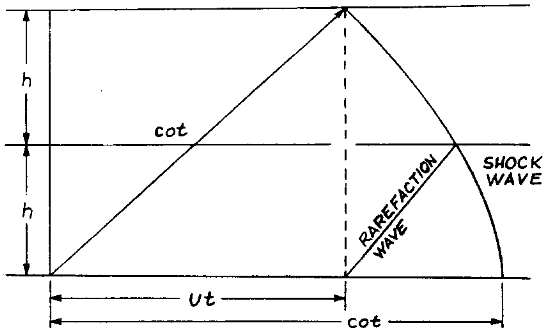


FIGURE 19

The distance $l_0$ between the beginning of the collision and the point of the emergence of the waves is

$$l_0 = Ut_0 = \frac{2hU}{\sqrt{c_0^2 - U^2}}. \tag{23}$$

The experimental verification of this fomula is difficult enough because of the local instability of the emergence of waves. Therefore, special experiments were carried out for the verification of the basic assumption on the role of the rarefaction wave. The scheme of these experiments is shown in Fig. 20. The experiments show that the steady waves emerge immediately behind the ledge if the size of the ledge is of the same order as the amplitude of the usual waves. If the ledge is absent, the waves appear in a random fashion (Fig. 21).

The observations suggest the existence of some oscillating mechanism in the vicinity of the point of contact, and the period of oscillations is determined only by the structure of the solution of the equations in this vicinity. This assumption is confirmed by the fact that the relations between $\lambda$ and $R$ and the parameters of the collision coincide (see Eqs. (16) and (17)).

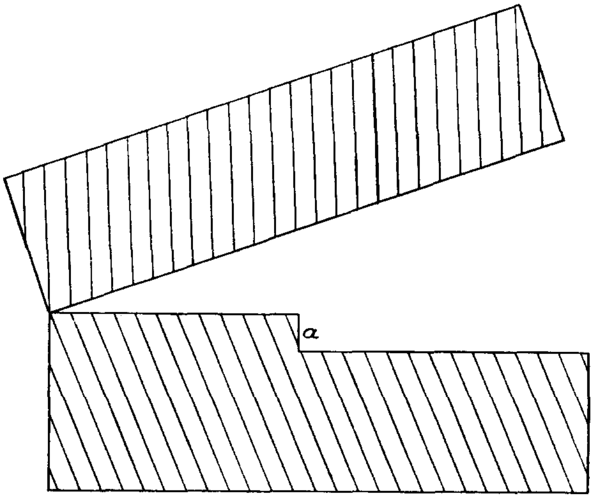


FIGURE 20

It is interesting to investigate the *width* of the rarefaction wave at the moment when it overtakes the point of contact. From previous investigations it follows that it is natural to expect that the rarefaction wave can be an initiator of waves if its width is comparable with the wave length.

The sound velocity increases behind the shock wave since the material is compressed. Let $c = c_0 + \delta c$. Let us assume $\delta c$ to be small and represent it in the form:

$$\delta c = \epsilon c_1(y). \tag{24}$$

In this case the situation shown in Fig. 19 will change, depending on the value of $y$ (Fig. 22).

The equations of the characteristic for the undisturbed flow are:

$$\partial\phi_0/\partial t + H_0(\phi_{0x}, \phi_y) = 0 \qquad H_0 = \sqrt{(\phi_{0x})^2 + (\phi_{0y})^2}. \tag{25}$$

Correspondingly, for the disturbed flow

$$\partial\phi/\partial t + H(y, \epsilon, \phi_x, \phi_y) = 0, \qquad \text{where} \quad \phi = \phi_0 + \epsilon\phi_\epsilon. \tag{26}$$

Since $\epsilon$ is small we can write

$$\frac{\partial\phi_\epsilon}{\partial t} + H_{\phi x}\frac{\partial\phi_\epsilon}{\partial x} + H_{\phi y}\frac{\partial\phi_\epsilon}{\partial y} + H_\epsilon = 0. \tag{27}$$

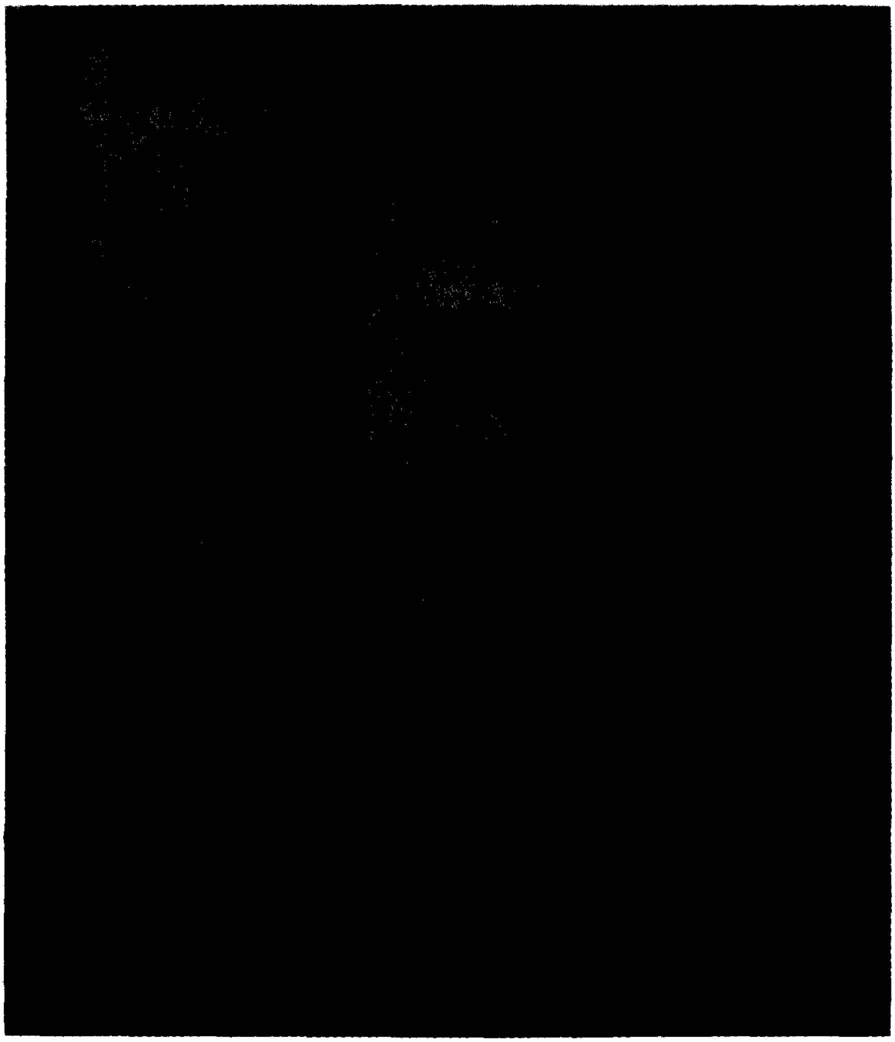

FIGURE 21

In our case

$$H = -c_0 \sqrt{(\phi_{0x})^2 + (\phi_{0y})^2} - \epsilon c_1(y) \sqrt{(\phi_{0x})^2 + (\phi_{0y})^2}$$

$$H_\epsilon = -c_1(y) \sqrt{(\phi_{0x})^2 + (\phi_{0y})^2}.$$

From the relations on the bicharacteristics

$$\frac{d\phi_\epsilon}{dy} = -\frac{c_1(y)}{c_0} \frac{\phi_{0x}^2 + \phi_{0y}^2}{\phi_{0y}} \tag{29}$$

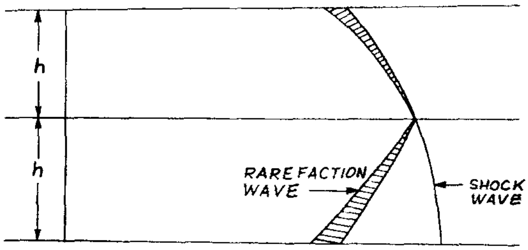


FIGURE 22

Since we are interested in the case of the wave propagating with the velocity $c_0$, and since the value of $c_1(y)$ is slightly different from the mean value of $\delta c/\epsilon = c_1 = \text{const}$, it is possible to obtain from Eq. (29):

$$\phi_\epsilon = -\frac{c_1}{c_0} \cdot 3h^2 \frac{x^2 + y^2}{y^2}. \tag{30}$$

Let $\delta\bar{x}$ be the *width* of the rarefaction wave. After perturbation with $\epsilon$

$$\delta\bar{x}/\epsilon = \phi_\epsilon/\phi_x .$$

From this relation and Eq. (23) we obtain the *width* of the rarefaction wave at $y = 2h$ at the moment of its arrival at the point of contact:

$$\frac{\delta\bar{x}}{l_0} = \frac{3}{16}\frac{\delta[c^2]}{U^2}. \tag{31}$$

It is possible to express $\delta[c^2]$ in terms of parameters of the collision. Assuming that $c_0/U \approx 2$ for a typical experiment,

$$\frac{\delta\bar{x}}{h} = \frac{3}{8}\frac{c_0/U}{\sqrt{(c_0/U)^2 - 1}} \sin\frac{\gamma}{2} \approx 0.86 \sin\frac{\gamma}{2}. \tag{32}$$

Comparing Eq. (32) with Eq. (17) it is easy to see that the relation $\lambda/\delta\bar{x}$ is changing from 1 to 2 at angles of collision between 5° and 10°, which are typical of most experiments.

## ACKNOWLEDGMENTS

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